Optimizing block-based maintenance under random machine usage

de Jonge, Bram; Jakobsons, Edgars

Published in:
European Journal of Operational Research

DOI:
10.1016/j.ejor.2017.07.051

Publication date:
2018

Optimizing block-based maintenance under random machine usage

Bram de Jonge[a,*], Edgars Jakobsons[b]

[a]*Department of Operations, Faculty of Economics and Business, University of Groningen, PO Box 800, 9700 AV Groningen, The Netherlands*
[b]*RiskLab, Department of Mathematics, ETH Zurich, Rämistrasse 101, 8092 Zurich, Switzerland*

Abstract

Existing studies on maintenance optimization generally assume that machines are either used continuously, or that times until failure do not depend on the actual usage. In practice, however, these assumptions are often not realistic. In this paper, we consider block-based maintenance optimization for a machine that is not used continuously and for which the usage is random. We propose to govern the random machine usage by a Markov switching (on-off), and present a method to determine the optimal maintenance interval. Various problem instances are considered, and the optimal maintenance intervals are compared with two benchmark intervals that result from the limiting cases with a very high and a very low switching frequency. Based on this analysis, we identify under what circumstances it is particularly important to take the properties of the usage pattern into account when scheduling maintenance.

Keywords: Maintenance, Block-based maintenance, Random usage

1. Introduction

Due to the ongoing automation of production processes and an increasingly competitive marketplace, effectively scheduled preventive maintenance activities become increasingly important (Alebrant Mendes and Ribeiro, 2014; Alsyouf, 2007). Two types of preventive maintenance policies can be distinguished: *condition-based* maintenance and *time-based* maintenance (Ahmad and Kamaruddin, 2012; De Jonge et al., 2017). Condition-based maintenance has the advantage that it leads to more effectively planned maintenance actions, because the condition of each machine is taken into account. However, condition monitoring is not always technically feasible, and the benefits may not outweigh the costs of implementing condition-based maintenance (e.g. monitoring equipment and software to analyze deterioration data). Therefore, many preventive maintenance actions in practice are still scheduled based on time. Also in the present paper, we focus on time-based preventive maintenance.

A further distinction can be made between *age-based* maintenance and *block-based* maintenance (Barlow and Hunter, 1960). Under both policies, corrective maintenance is carried out upon failure of the unit. The age-based maintenance policy performs preventive maintenance when the unit reaches a specified age T. The age is set back to zero at failure. Under the block-based maintenance policy preventive maintenance is performed at fixed times T, $2T$, $3T$, ... In contrast to the age-based maintenance policy, the preventive maintenance schedule is not updated when a failure occurs. The disadvantage of block-based maintenance is that preventive maintenance is sometimes performed shortly after corrective maintenance. The main advantages, on the other hand, are the easier planning, as it is known in advance when preventive

[*]Corresponding author.
Email addresses: `b.de.jonge@rug.nl` (Bram de Jonge), `edgars.jakobsons@cantab.net` (Edgars Jakobsons)

maintenance will be performed, and the clustered maintenance actions, if the same block-based policy is used for multiple units. The latter is often beneficial due to economic dependencies between units. In this paper, we consider block-based maintenance for a single unit. The analysis, however, is also immediately applicable to systems with multiple identical units. Other recent studies on block-based maintenance for single units include Borrero and Akhavan-Tabatabaei (2013); Khojandi et al. (2014); Zhang et al. (2014); Zhao et al. (2015).

Existing studies on block-based maintenance optimization generally assume either that machines are used continuously, or that times until failure do not depend on the actual machine usage; see Coria et al. (2015), Godoy et al. (2014), Khojandi et al. (2014), and Park and Pham (2012) for recent examples. In many practical situations, however, these assumptions are unrealistic. Machines are generally not used continuously (Jack et al., 2009; Murthy et al., 1995) and degradation is likely to depend on the actual usage (Finkelstein, 2004; Tinga, 2010). In this paper, we consider the block-based maintenance policy for a machine that is not used continuously and for which the future usage is stochastic. We govern the random machine usage by a Markov switching (on-off). We provide a method for computing the optimal maintenance interval that minimizes the long-run cost rate and analyze the obtained results. Furthermore, we compare these with two benchmark intervals that correspond to two limiting cases, respectively with very frequent and very infrequent switching. These limiting cases can be analyzed based on the standard block-based maintenance model. Based on the analysis, we provide insights about the effect of the usage pattern on the optimal preventive maintenance interval, and point out under what circumstances it is especially important to take the properties of the usage pattern into account when applying the block-based maintenance policy.

The remainder of this paper is organized as follows. In Section 2, we formulate the problem mathematically. In Section 3, we analyze the model, distinguish the two limiting cases, and describe a numerical method for solving the general model. In Section 4, we solve numerous instances of the problem and use the results to provide interesting insights. We end in Section 5 with conclusions and directions for future research.

2. Problem description

We consider block-based maintenance (Barlow and Hunter, 1960; Barlow and Proschan, 1965) for a single machine with a random usage pattern. Under the block-based maintenance policy, preventive maintenance is performed periodically every T time units, and corrective maintenance is carried out when the machine fails in between preventive maintenance actions. These corrective maintenance actions do not influence the preventive maintenance schedule. Both preventive and corrective maintenance are assumed to require a negligible amount of time and to make the machine as-good-as-new. Furthermore, preventive maintenance is assumed to be less expensive than corrective maintenance, implying that performing preventive maintenance might be beneficial when scheduled effectively. These are common assumptions in maintenance modeling, see for example De Jonge et al. (2015a); Jiang and Jardine (2007); Sheu and Zhang (2013); Zitrou et al. (2013). We normalize the cost of a corrective maintenance action to 1, and denote the cost of a preventive maintenance action by $c < 1$. The aim is to determine the preventive maintenance interval T that minimizes the long-run cost rate.

The random machine usage is governed by a Markov switching. The machine is alternately active and idle, and independent exponential durations are used to model the lengths of these periods. Active periods have exponentially distributed lengths with parameter α_1, idle periods have exponentially distributed lengths with parameter α_0. The corresponding exponential distribution functions are denoted by F_1 and F_0, respectively. Due to the memorylessness of the exponential distribution, both the instantaneous rate of switching off the machine when it is active and the instantaneous rate of switching it on when it is idle are constant over time. Besides this reasonable property, exponential durations are also adopted for analytical tractability.

We denote the length of the i^{th} active period by X_i^1, and the length of the i^{th} idle period by X_i^0. Because of the exponential active and idle periods, the usage of the machine can be described by a continuous-time Markov chain $(S_t)_{t\geq 0}$ on states $\{0,1\}$, where 1 refers to active and 0 refers to idle. Furthermore, since the active periods have mean length $1/\alpha_1$ and the idle periods have mean length $1/\alpha_0$, the usage rate (i.e. the fraction of time the machine is active), denoted by ρ, is given by

$$\rho = \frac{\frac{1}{\alpha_1}}{\frac{1}{\alpha_1}+\frac{1}{\alpha_0}} = \frac{\alpha_0}{\alpha_0+\alpha_1}. \tag{1}$$

The lifetime distribution function of the machine is denoted by F_W. It is assumed that the machine only deteriorates when it is active, i.e., only the active periods constitute the time until failure. Thus, for a random lifetime W, a failure occurs when the total time the machine has been active reaches W. We refer to the age of the machine as the total time that the machine has been active since the last maintenance action (either preventive or corrective). The likelihood of failure thus depends on the age of the machine.

Suppose the machine is in the "new" condition at time $t=0$. The time until failure Y can then be expressed as a sum of the lifetime W and a compound process that depends on W:

$$Y = W + \sum_{i=1}^{N} X_i^0, \quad \text{where} \quad N = \min\left\{n : \sum_{i=1}^{n} X_i^1 > W\right\} - S_0. \tag{2}$$

In Figure 1, a clarification of this formula is provided. The first diagram illustrates the case that the machine is active at $t=0$ (i.e. $S_0=1$), and the second diagram the case it is idle (i.e. $S_0=0$). In both cases the running time until failure W is reached during the third active period. In addition, N denotes the number of idle periods that are included in the time until failure. In this case, $N=3$ if the machine is idle at time $t=0$, and $N=2$ if it is active.

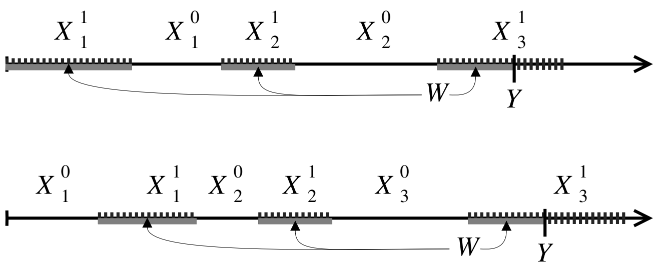

Figure 1: Illustration of the time until failure; beginning with an active (top) or idle (bottom) machine.

The machine is assumed to be in the "new" condition both after preventive and after corrective maintenance. The preventive maintenance actions are performed regardless of the state of the machine, hence the machine is active with probability ρ at a preventive maintenance action, and idle with probability $1-\rho$. Failure, on the other hand, can only occur when the machine is active, implying that the machine is always in the active state immediately after a corrective maintenance action. Denoting the times between failures after a preventive maintenance action by $(Y_i)_{i\geq 1}$, it follows that Y_1 is distributed as Y in (2), where $S_0=1$ with probability ρ and $S_0=0$ with probability $1-\rho$, whereas $Y_2, Y_3, \ldots$ are distributed as Y with $S_0=1$ with certainty. It follows that the counting process of the number of failures after a preventive maintenance action is a *delayed renewal process* with holding times $(Y_i)_{i\geq 1}$; see Resnick (2013) for details on renewal processes. We denote the corresponding renewal function by $m(t)$.

We let $C(T,t)$ denote the total costs over a period with length t when the block-based maintenance policy with preventive maintenance interval T is used. Our aim is to minimize the long-run cost rate $\eta(T) = \lim_{t\to\infty} C(T,t)/t$. This cost rate can be calculated based on standard renewal theory with the preventive maintenance actions as renewal points. Recall that preventive maintenance is performed every T time units at cost c, and corrective maintenance is carried out at cost 1 at each failure between preventive maintenance actions. By referring to the time between two preventive maintenance actions as a cycle, the expected cost per cycle is $c + 1\cdot m(T)$ and the (fixed) cycle length is T. It follows from standard renewal theory that the long-run cost rate as a function of the maintenance interval T, denoted by $\eta(T)$, is given by

$$\eta(T) = \lim_{t\to\infty} \frac{C(T,t)}{t} = \frac{c + m(T)}{T}. \tag{3}$$

The maintenance interval $T > 0$ that minimizes this cost rate is the optimal maintenance interval T_{opt}. Note that, by the strong law of large numbers, the cost rate $\eta(T)$ converges to $\rho/\mathrm{E}[W]$ as the maintenance interval T tends to ∞. If $\eta(T) > \rho/\mathrm{E}[W]$ for all $T > 0$, then it is optimal to perform no preventive maintenance at all; in this case we have $T_{\text{opt}} = \infty$ and $\eta(\infty) = \rho/\mathrm{E}[W]$.

3. Analysis

Analytic solutions for renewal functions of renewal processes are available only for special cases of the holding time distribution, and in our case the distributions of the holding times Y_1, Y_2, ... are not even known in a closed form. In this section, we will first consider two limiting cases that can be analyzed using the renewal function of the lifetime distribution. Thereafter, we will represent the general case by a set of integral equations, and provide a numerical procedure that can be used to compute the cost rate $\eta(T)$.

3.1. Limiting cases

Two limiting cases can be analyzed based on the usage rate ρ as given by (1) and the renewal function $m_W(t)$ of the lifetime distribution function $F_W(t)$. The first limiting case is that of a very high switching frequency. In this case, the machine is active approximately for time period ρT between any two consecutive preventive maintenance actions, and failures can only occur during this time period. The expected number of failures in between two preventive maintenance actions can thus accurately be approximated by $m_W(\rho T)$, and the cost rate by

$$\eta_{\text{freq}}(T) = \frac{c + m_W(\rho T)}{T}.$$

We denote the corresponding optimal maintenance interval by T_{freq}. If, on the other hand, the switching frequency is very low (rare switching), it becomes very likely that the machine is either entirely active or entirely idle in between two consecutive preventive maintenance actions. The probability of being active is ρ, implying that the expected failure cost $m_W(T)$ is only incurred with probability ρ. The cost rate can thus accurately be approximated by

$$\eta_{\text{rare}}(T) = \frac{c + \rho m_W(T)}{T}.$$

The corresponding optimal maintenance interval will be denoted by T_{rare}. Various methods exist for numerically evaluating the renewal function of a lifetime distribution. One could, for example, use the first few convolutions of the lifetime distribution to obtain an approximation, see for example Gertsbakh (2000). Because the number of failures in between preventive maintenance actions is generally small for reasonable relative preventive maintenance costs c and maintenance intervals T, this method provides very accurate approximations.

3.2. General solution

In order to represent the general case of the problem by a set of integral equations, we define functions $g_0(t,w)$ and $g_1(t,w)$ that give the expected number of failures during time period t, starting with a machine of age w, being idle or active, as indicated by the subscripts 0 and 1, respectively. The system of integral equations is obtained by conditioning on "the first thing that happens". If the machine is idle, this first event can only be the machine being turned on. Thus, to express $g_0(t,w)$ in terms of g_1, we should realize that the machine can be turned on at any time s during time period t. This results in an active machine, still with age w, and with time $t-s$ left. If the machine is active, the first event is either the machine being turned off or failing. Thus, $g_1(t,w)$ should be expressed both in terms of g_0 and g_1 itself. If the first event is a failure of the machine, the number of failures will be increased by 1, and the result is a machine with age 0. Hence, by writing $\bar{F} = 1 - F$ for the reliability function corresponding to distribution function F, it follows that

$$\begin{aligned} g_0(t,w) &= \int_0^t g_1(t-s,w)\,\mathrm{d}F_0(s), \\ g_1(t,w) &= \frac{1}{\bar{F}_W(w)} \left(\int_0^t g_0(t-s,w+s)\bar{F}_W(w+s)\,\mathrm{d}F_1(s) \right. \\ &\quad \left. + \int_0^t (1+g_1(t-s,0))\bar{F}_1(s)\,\mathrm{d}F_W(w+s) \right), \end{aligned} \tag{4}$$

with boundary conditions

$$\begin{aligned} g_0(0,w) &= 0, \\ g_1(0,w) &= 0. \end{aligned}$$

Since at the start of a cycle the machine is active with probability $\rho = \alpha_1/(\alpha_0+\alpha_1)$, and idle with probability $1-\rho$, the renewal function $m(T)$ in (3), i.e., the expected number of failures in between two preventive maintenance actions, equals

$$m(T) = (1-\rho)g_0(T,0) + \rho g_1(T,0). \tag{5}$$

Hence, solving the system of equations (4) enables us to calculate the cost rate (3), from which the optimal maintenance interval T_{opt} can be found.

We aim to solve the system of equations (4) numerically, by evaluating the integrals using the midpoint rule from numerical analysis. Since the functions F_0, F_1, and F_W are known, whereas the functions g_0 and g_1 are unknown, it is preferable to integrate the known functions with respect to the change in g_0 and g_1, which are computed on a grid. We rewrite this system of equations using an approach similar to that proposed by Xie (1989) for solving (one-dimensional) renewal-type integral equations. In particular, we use integration by parts, substitution $u = t-s$, the fact that $F_0(0) = F_1(0) = 0$, and the property $\mathrm{d}F_W(w+s) = -\,\mathrm{d}\bar{F}_W(w+s)$ to transform the system of equations (4) into

$$\begin{aligned} g_0(t,w) &= \int_0^t F_0(t-u)\,\mathrm{d}g_1(u,w), \\ g_1(t,w) &= \int_0^t F_1(t-u)\,\mathrm{d}\Big(g_0(u,w+t-u)\bar{F}_W(w+t-u)/\bar{F}_W(w)\Big) \\ &\quad + \int_0^t \bar{F}_1(t-u)\,\mathrm{d}\Big(\bar{F}_W(w+t-u)/\bar{F}_W(w)\Big) + g_1(t,0) \\ &\quad - \int_0^t \bar{F}_W(w+t-u)/\bar{F}_W(w)\,\mathrm{d}\Big(g_1(u,0)\bar{F}_1(t-u)\Big). \end{aligned} \tag{6}$$

To solve the transformed system of equations (6), a maximum considered preventive maintenance interval $t_{\max}$ is chosen. Given a time period t until the next preventive maintenance action, the age w of the machine is at most $t_{\max} - t$, therefore the considered domain is $\{(t, w) : 0 \leq t \leq t_{\max}, 0 \leq w \leq t_{\max} - t\}$. We discretize this domain with a regular mesh of stepsize $\delta = t_{\max}/N$ and N steps. With the aim of implementing a computer program to solve these equations, by abuse of notation we write $f(i)$ instead of $f(i\delta)$ for all functions f, and approximate the integrals in (6) by sums according to the midpoint rule:

$$
\begin{aligned}
g_0(i,j) &= \sum_{k=1}^{i} F_0(i-k+\tfrac{1}{2})(g_1(k,j) - g_1(k-1,j)),\\
g_1(i,j) &= \sum_{k=1}^{i} F_1(i-k+\tfrac{1}{2})\Big(g_0(k,j+i-k)\bar{F}_W(j+i-k)/\bar{F}_W(j)\\
&\qquad\qquad - g_0(k-1,j+i-k+1)\bar{F}_W(j+i-k+1)/\bar{F}_W(j)\Big)\\
&\quad + \sum_{k=1}^{i} \bar{F}_1(i-k+\tfrac{1}{2})\Big(\bar{F}_W(j+i-k)/\bar{F}_W(j) - \bar{F}_W(j+i-k+1)/\bar{F}_W(j)\Big) + g_1(i,0)\\
&\quad - \sum_{k=1}^{i} \bar{F}_W(j+i-k+\tfrac{1}{2})/\bar{F}_W(j)\Big(g_1(k,0)\bar{F}_1(i-k) - g_1(k-1,0)\bar{F}_1(i-k+1)\Big).
\end{aligned}
\tag{7}
$$

For $i \geq 1$, $g_0(i,j)$ and $g_1(i,j)$ can be expressed in terms of $g_0(k,\cdot)$, $g_1(k,\cdot)$, $k < i$, $g_0(i,0)$ and $g_1(i,0)$. To obtain a system of linear equations in the form "$Ag = b$", for $i \geq 1$ and $j = 0$, we rewrite (7) as

$$
\begin{aligned}
g_0(i,0) - F_0(\tfrac{1}{2})g_1(i,0) &= \sum_{k=1}^{i-1} F_0(i-k+\tfrac{1}{2})(g_1(k,0) - g_1(k-1,0)) - F_0(\tfrac{1}{2})g_1(i-1,0),\\
-F_1(\tfrac{1}{2})g_0(i,0) + \bar{F}_W(\tfrac{1}{2})g_1(i,0) &= \sum_{k=1}^{i-1} F_1(i-k+\tfrac{1}{2})\Big(g_0(k,i-k)\bar{F}_W(i-k)\\
&\qquad\qquad - g_0(k-1,i-k+1)\bar{F}_W(i-k+1)\Big)\\
&\quad - F_1(\tfrac{1}{2})g_0(i-1,1)\bar{F}_W(1)\\
&\quad + \sum_{k=1}^{i} \bar{F}_1(i-k+\tfrac{1}{2})\Big(\bar{F}_W(i-k) - \bar{F}_W(i-k+1)\Big)\\
&\quad - \sum_{k=1}^{i-1} \bar{F}_W(i-k+\tfrac{1}{2})\\
&\qquad\qquad \cdot\Big(g_1(k,0)\bar{F}_1(i-k) - g_1(k-1,0)\bar{F}_1(i-k+1)\Big)\\
&\quad + \bar{F}_W(\tfrac{1}{2})g_1(i-1,0)\bar{F}_1(1).
\end{aligned}
\tag{8}
$$

For $i \geq 1$ and $j \geq 1$, it can be written as

$$
\begin{aligned}
g_0(i,j) - F_0(\tfrac{1}{2})g_1(i,j) &= \sum_{k=1}^{i-1} F_0(i-k+\tfrac{1}{2})(g_1(k,j) - g_1(k-1,j)) - F_0(\tfrac{1}{2})g_1(i-1,j), \\
-F_1(\tfrac{1}{2})g_0(i,j) + g_1(i,j) &= \sum_{k=1}^{i-1} F_1(i-k+\tfrac{1}{2})\Big(g_0(k,j+i-k)\bar{F}_W(j+i-k)/\bar{F}_W(j) \\
&\qquad\qquad - g_0(k-1,j+i-k+1)\bar{F}_W(j+i-k+1)/\bar{F}_W(j)\Big) \\
&\quad - F_1(\tfrac{1}{2})g_0(i-1,j+1)\bar{F}_W(j+1)/\bar{F}_W(j) \\
&\quad + \sum_{k=1}^{i} \bar{F}_1(i-k+\tfrac{1}{2}) \\
&\qquad\qquad \cdot \Big(\bar{F}_W(j+i-k)/\bar{F}_W(j) - \bar{F}_W(j+i-k+1)/\bar{F}_W(j)\Big) \\
&\quad + g_1(i,0) - \sum_{k=1}^{i} \bar{F}_W(j+i-k+\tfrac{1}{2})/\bar{F}_W(j) \\
&\qquad\qquad \cdot \Big(g_1(k,0)\bar{F}_1(i-k) - g_1(k-1,0)\bar{F}_1(i-k+1)\Big).
\end{aligned} \tag{9}
$$

Combining (8) and (9) with the boundary conditions $g_0(0,j) = 0$ and $g_1(0,j) = 0$ for $j = 0,1,2,\ldots$, we can solve $g_0(i,j)$ and $g_1(i,j)$ iteratively for each $i = 1,\ldots,N$ and each $j = 0,\ldots,N-i$, in this order.

4. Results

We use the approach described in the previous section to investigate how the optimal maintenance interval T_{opt} and the corresponding cost rate $\eta(T_{\text{opt}})$ are affected by the parameter values of the model. We also compare the optimal cost rate with the (actual) cost rates when using the frequent and rare switching approximations T_{freq} and T_{rare} and identify under what circumstances it is especially important to take the actual usage pattern into account when choosing the maintenance interval.

We use the Weibull distribution to model the lifetimes of the machine, and remark that other suitable distributions (e.g. gamma and lognormal) lead to similar insights. The Weibull distribution is the most commonly used distribution to model lifetimes and has been found to provide a good description for many types of lifetime data (Lawless, 2002; Rinne, 2008). Recent studies that also adopt a Weibull lifetime distribution include De Jonge et al. (2015b); Xia et al. (2015); Xu and Cao (2015); Zhou et al. (2015). The Weibull distribution has a shape parameter k and a scale parameter λ; the distribution function F_W is given by

$$F_W(t) = 1 - \mathrm{e}^{-(t/\lambda)^k}, \quad t \geq 0.$$

Only shape parameters $k > 1$ correspond to an increasing failure rate, implying that preventive maintenance can be beneficial. We will therefore not consider cases with $k \leq 1$. Without loss of generality (i.e. by adjusting the time scale), we fix the mean $\mathrm{E}[W] = \lambda\Gamma(1+1/k)$ of the Weibull lifetime distribution to 1.

Figure 2 shows the cost rate $\eta(T)$ as a function of the maintenance interval T for a Weibull shape parameter $k = 5$, a relative preventive maintenance cost $c = 0.1$, and usage rates $\rho = 1/3$, $1/2$, and $2/3$. Various switching frequencies are considered; the dotted lines correspond to the limiting cases of “frequent” and “rare” switches. Note that, by specifying ρ and α_1, if follows from (1) that $\alpha_0 = \alpha_1\rho/(1-\rho)$ is fixed. The differences between the two limiting cases turn out to be significant, and the cost rate function gradually transforms from the rare switching case to the frequent switching case as the switching frequency increases.

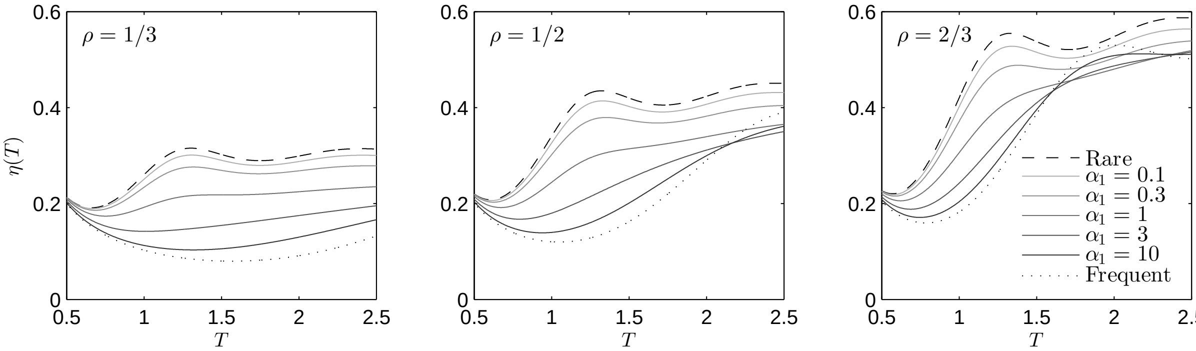

Figure 2: Cost rate $\eta(T)$ as a function of the maintenance interval T, for usage rates $\rho = 1/3$, $1/2$, and $2/3$, for various switching frequencies (specified by α_1), and for $k = 5$ and $c = 0.1$.

Another important observation from Figure 2 is that the graphs of the cost rate $\eta(T)$ are not necessarily unimodal and might have multiple local minima. This implies that the first local minimum is not necessarily the global minimum, and that we should evaluate $\eta(T)$ on a sufficiently large domain $[0, t_{\max}]$ to make sure that we find the global minimum. This phenomenon is illustrated by Figure 3, which is based on parameter values $k = 5$, $c = 0.25$, $\alpha_0 = 0.2$, and $\alpha_1 = 0.3$. The left panel of this figure, which depicts $\eta(T)$ up to $t_{\max} = 2.3$, shows that the cost rate at the second local minimum is a lower than at the first local minimum. However, by evaluating $\eta(T)$ for even higher maintenance intervals T, see the right panel of the figure, it turns out that the global minimum is attained at $T_{\text{opt}} \approx 5.09$. Moreover, in some cases it may be optimal to perform no preventive maintenance at all, as we can see e.g. for the rare switching case in Figure 3 ($T_{\text{rare}} = \infty$).

From our numerical experiments, we observe that optimal preventive maintenance intervals are especially large for low usage rates ρ and for high relative preventive maintenance costs c. In our analysis, we computed the cost rate $\eta(T)$ up to $t_{\max} = (1 + 5c)/\rho$, which, visually, seems to be sufficient for the considered cases. In cases where the cost rate $\eta(T)$ over the entire interval $[0, t_{\max}]$ was higher than the asymptotic cost rate $\rho/\mathrm{E}[W] = \rho$, we conclude that it is optimal not to perform any preventive maintenance at all (i.e., that $T_{\text{opt}} = \infty$).

Figure 4 shows the optimal maintenance interval T_{opt} and the corresponding cost rate $\eta(T_{\text{opt}})$ as a function of the usage rate ρ, and for various switching frequencies. Again, $k = 5$ and $c = 0.1$ are considered. Obviously, when the usage rate ρ increases (all else unchanged), preventive maintenance needs to be performed more often. The optimal maintenance interval T_{opt} is therefore decreasing in ρ. And, since more intensive usage leads to higher maintenance costs, also the optimal cost rate $\eta(T_{\text{opt}})$ increases with ρ. The differences between the switching frequencies vanish when the usage rate ρ converges to 1.

When the switching frequency of the machine decreases, the usage over time becomes less predictable, and the occurrence of longer active periods becomes more likely. In order to avoid failures during such long active periods, the optimal maintenance interval T_{opt} becomes shorter. Furthermore, preventive maintenance becomes less effective if there is more randomness, resulting in higher cost rates when the switching frequency is low.

Without our analysis, one might resort to using one of the maintenance intervals T_{freq} or T_{rare} of the two limiting cases. Our study is therefore particularly relevant in situations where

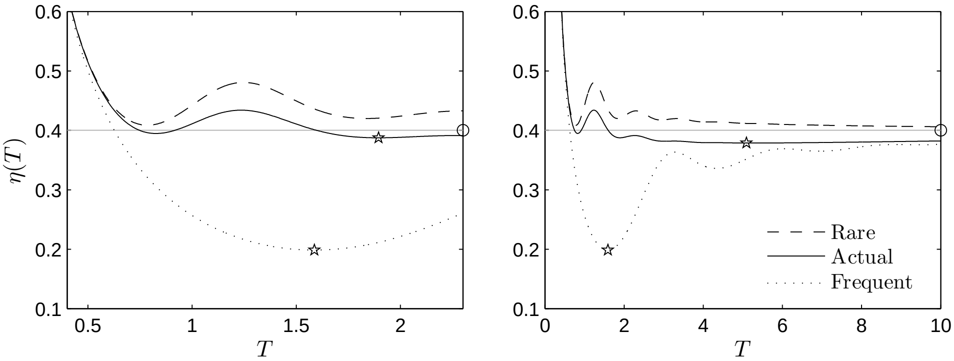

Figure 3: Cost rate $\eta(T)$ as a function of the maintenance interval T, for $k = 5$, $c = 0.25$, $\alpha_0 = 0.2$, $\alpha_1 = 0.3$; stars mark the locations of minima on the respective considered domains for T, circles denote that for the rare switching approximation $T_{\text{rare}} = \infty$, and $\eta(T_{\text{rare}}) = \rho = 0.4$ (gray line).

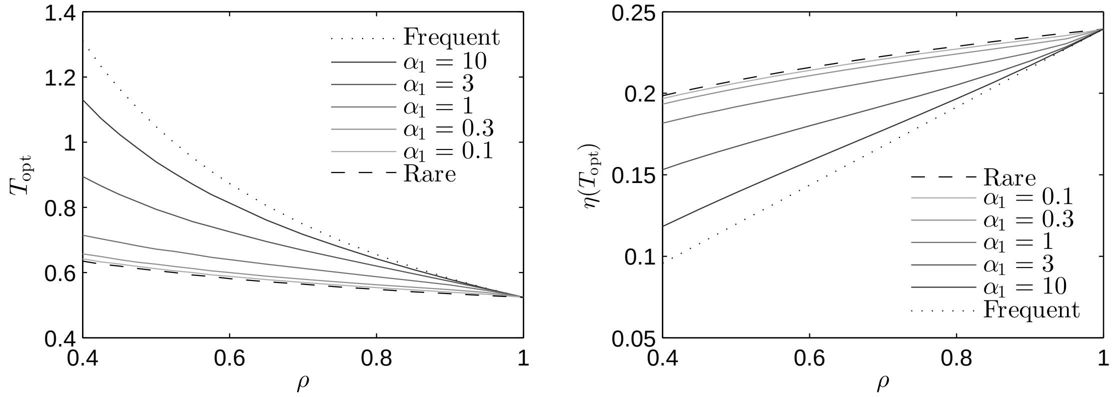

Figure 4: Optimal maintenance interval T_{opt} *(left panel)* and corresponding optimal cost rate $\eta(T_{\text{opt}})$ *(right panel)* as a function of the usage rate ρ, for various switching frequencies (specified by α_1) and for $k = 5$ and $c = 0.1$.

the cost rate $\eta(T_{\text{opt}})$ under the optimal maintenance interval T_{opt} is significantly lower than the cost rates $\eta(T_{\text{freq}})$ and $\eta(T_{\text{rare}})$ under the limiting policies. Figure 5 shows the relative cost savings from using the optimal maintenance interval T_{opt} instead of using the (suboptimal) maintenance intervals T_{freq} and T_{rare}, based on the limiting approximations. A base case with parameter values $k = 5$, $c = 0.1$, $\rho = 0.5$, and $\alpha_1 = 3$ is considered, and these parameter values are then varied one at a time in the four panels of the figure. We note that, without this analysis, it is not clear which approximation is closer to the actual situation, and, as a consequence, the worse of the two approximations T_{freq} and T_{rare} might be selected. Therefore, both lines in the panels of Figure 5 are relevant, not only the minimum of these.

For low values of the shape parameter k of the Weibull lifetime distribution, the failure rate increases slowly and the cost rate is quite flat, therefore there is little difference in costs between the various maintenance intervals. These differences become quite significant for higher values of k. This is especially the case when the frequent switching approximation is used. Preventive maintenance is then regularly performed too late, and the high corrective maintenance cost is often incurred. A similar effect occurs for very low preventive maintenance costs c: performing preventive maintenance too early is better than being too late. This effect reverses when c increases: being too late occasionally is then preferred to being early too often. The discontinuity in the second panel of Figure 5 is caused by the fact that, under the rare switching approximation, it becomes optimal not to perform any preventive maintenance if c exceeds approximately 0.3 (i.e., $T_{\text{rare}} = \infty$); for the actual usage pattern, this occurs closer to $c = 0.5$.

When the usage rate ρ decreases (all else unchanged), the idle periods become longer and the active periods become sporadic. This makes the block-based maintenance policy less effective; the optimal maintenance interval is therefore quite large. Since the frequent switching approximation also selects a relatively large maintenance interval, the performance of this approximation is satisfactory. The much smaller maintenance interval selected by the rare switching approximation, on the other hand, is considerably more expensive. For moderate usage rates, there is a significant gap between the performance of the optimal maintenance interval and the two approximations. When the usage rate tends to 1, the difference between the optimal policy and the limiting approximations vanishes, as expected. The last panel of Figure 5 shows that the rare switching approximation performs better when the switching frequency is relatively low, whereas the frequent switching approximation performs better for higher switching frequencies, in agreement with the intuition.

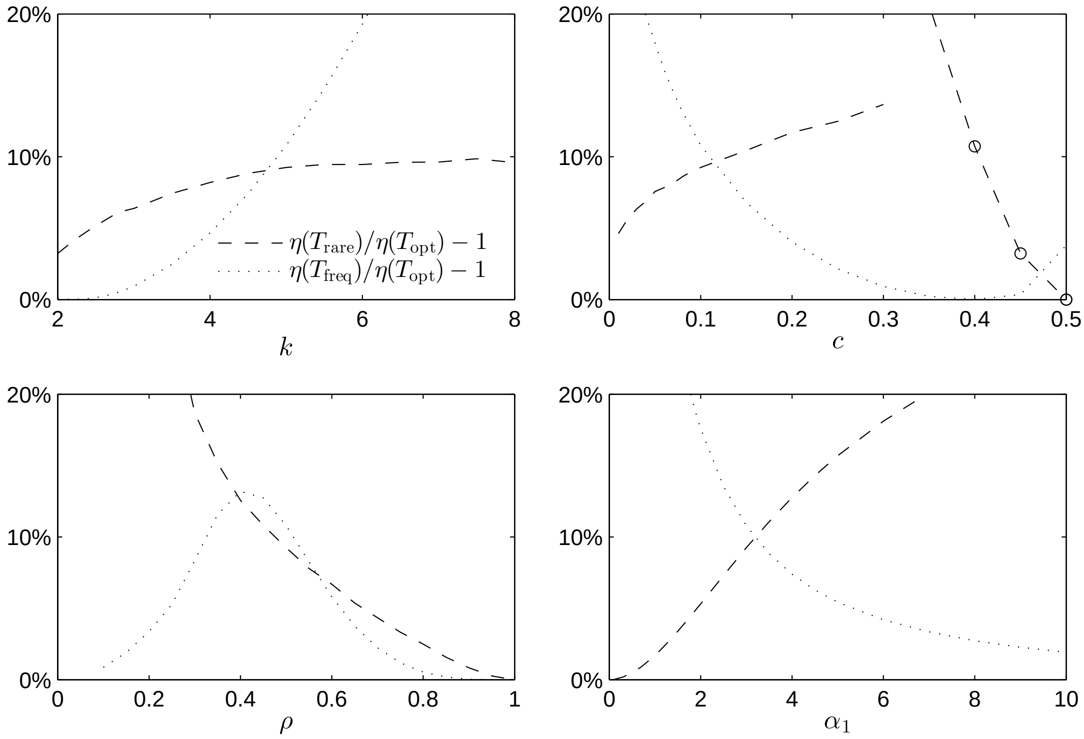

Figure 5: Relative cost saving from using the optimal maintenance interval T_{opt} instead of using the approximations T_{rare} or T_{freq}. The base case is $k = 5$, $c = 0.1$, $\rho = 0.5$, $\alpha_1 = 3$, and in each of the four panels one of the parameters is varied. The circles in the top-right panel indicate cases where $T_{\text{rare}} = \infty$.

A final note is that, even though the machine has an increasing failure rate and the preventive maintenance cost is lower than the corrective maintenance cost, the optimal block-based maintenance policy can be to perform no preventive maintenance at all (i.e., $T_{\text{opt}} = \infty$). In the example plotted in Figure 3, this would be optimal for very low switching frequencies. In general, we observed three situations in which this could be the case; namely (i) when the failure rate increases slowly (low k), (ii) when the relative preventive maintenance cost c is high, and (iii) when the usage rate ρ is low. In these cases, either the moment of failure is too difficult to predict, or the cost saving of preventive maintenance compared to corrective maintenance is too small.

5. Conclusions and future extensions

We have considered block-based maintenance scheduling for a single machine. Furthermore, while existing research generally assumes either that machines are used continuously, or that deterioration of the machine is independent of the actual machine usage, we have considered the often more realistic setting of a machine that is not used continuously and for which the actual usage over time is difficult to predict. Moreover, the machine only deteriorates when it is used. A property of the block-based maintenance policy is that preventive maintenance actions have to be planned in advance, at a regular interval with length T. Corrective maintenance is only performed when a failure occurs between preventive maintenance actions; this does not influence the preventive maintenance schedule. The aim is to determine the optimal maintenance interval under a random machine usage.

We have proposed a model in which the machine usage is characterized by active periods alternating with idle periods, both of random lengths. We have developed a method to numerically calculate the long-run cost rate as a function of the maintenance interval, enabling us to determine the optimal maintenance interval. After implementing this method, we have considered numerous problem instances, and analyzed the effects of changing the parameter values of the model. Furthermore, we have considered two approximating maintenance intervals that are based on frequent and rare machine switching, respectively. These intervals can be determined based on the standard block-based maintenance model. The performance of the optimal maintenance interval has been compared with these two approximations.

The optimal maintenance interval is obviously decreasing as the usage rate increases, i.e., preventive maintenance is performed more often if the machine is used more. Furthermore, for fixed usage rates, a higher switching frequency implies a more stable usage, and therefore a larger maintenance interval. Without our analysis, one might rely on the two approximating maintenance intervals. The importance of this study is emphasized by the observation that, for a wide variety of cases, the costs of using these approximations are significantly higher than the costs under the optimal maintenance policy.

Various opportunities exist for future research in this area. First, while our solution method can be used for any lifetime distribution, the assumption of having active and idle periods with exponential lengths is crucial. Without the memorylessness of the exponential distribution, the time that the machine has already been active or idle should also be taken into account. This adds a third dimension to the system of integral equations and makes their analysis more burdensome. Future research could study this generalization analytically, or one could resort to simulations (De Jonge, 2015). Another extension would be to consider more than the two machine speeds (on and off) that we consider in this paper. This would require additional assumptions for modeling the random machine usage, and for the relation between the machine speed and the deterioration rate.

Taking the actual machine usage into account is also important when condition-based maintenance policies are applied. In such settings, preventive maintenance is commonly initiated when a certain threshold deterioration level is reached, and is performed after a required lead time (e.g. Grall et al., 2006; Saassouh et al., 2007). Existing studies assume that the machine is used continuously during this lead time. However, when this is not the case, it is to be expected that it is more efficient to set the preventive maintenance threshold at a higher level.

Finally, in our study, we have assumed that the machine usage is determined externally and that the maintenance strategy has to be adapted to this usage pattern. However, when there is some flexibility in scheduling the machine usage, the combined optimization of maintenance actions and machine usage is also of interest. When, for instance, the likelihood of failure becomes too high, one could consider delaying operations by temporarily switching off the equipment until the next preventive maintenance action has been carried out.

Acknowledgement

E. Jakobsons acknowledges the financial support from RiskLab Switzerland and the Swiss Finance Institute.

References

Ahmad, R., Kamaruddin, S., 2012. An overview of time-based and condition-based maintenance in industrial applications. Computers & Industrial Engineering 63 (1), 135–149.

Alebrant Mendes, A., Ribeiro, J. L. D., 2014. Establishment of a maintenance plan based on quantitative analysis in the context of RCM in a JIT production scenario. Reliability Engineering and System Safety 127, 21–29.

Alsyouf, I., 2007. The role of maintenance in improving companies' productivity and profitability. International Journal of Production Economics 105 (1), 70–78.

Barlow, R. E., Hunter, L., 1960. Optimum preventive maintenance policies. Operations Research 8 (1), 90–100.

Barlow, R. E., Proschan, F., 1965. Mathematical Theory of Reliability. Wiley.

Borrero, J. S., Akhavan-Tabatabaei, R., 2013. Time and inventory dependent optimal maintenance policies for single machine workstations: An MDP approach. European Journal of Operational Research 3 (228), 545–555.

Coria, V. H., Maximov, S., Rivas-Dávalos, F., Melchor, C. L., Guardado, J. L., 2015. Analytical method for optimization of maintenance policy based on available system failure data. Reliability Engineering and System Safety 135, 55–63.

De Jonge, B., 2015. A discrete event simulation design for block-based maintenance planning under random machine usage. In: Podofillini, L., Sudret, B., Stojadinovic, B., Zio, E., Kroger, W. (Eds.), Safety and Reliability of Complex Engineered Systems (ESREL 2015). CRC Press, Boca Raton, p. 398.

De Jonge, B., Dijkstra, A. S., Romeijnders, W., 2015a. Cost benefits of postponing time-based maintenance under lifetime distribution uncertainty. Reliability Engineering and System Safety 140, 15–21.

De Jonge, B., Klingenberg, W., Teunter, R. H., Tinga, T., 2015b. Optimum maintenance strategy under uncertainty in the lifetime distribution. Reliability Engineering and System Safety 133, 59–67.

De Jonge, B., Teunter, R. H., Tinga, T., 2017. The influence of practical factors on the benefits of condition-based maintenance over time-based maintenance. Reliability Engineering and System Safety 158, 21–30.

Finkelstein, M., 2004. Alternative time scales for systems with random usage. IEEE Transactions on Reliability 53 (2), 261–264.

Gertsbakh, I., 2000. Reliability Theory, With Applications to Preventive Maintenance. Springer, Berlin, Germany.

Godoy, D. R., Pascual, R., Knights, P., 2014. A decision-making framework to integrate maintenance contract conditions with critical spares management. Reliability Engineering and System Safety 131, 102–108.

Grall, A., Dieulle, L., Bérenguer, C., Roussignol, M., 2006. Asymptotic failure rate of a continuously monitored system. Reliability Engineering and System Safety 91 (2), 126–130.

Jack, N., Iskandar, B. P., Murthy, D. N. P., 2009. A repair-replace strategy based on usage rate for items sold with a two-dimensional warranty. Reliability Engineering and System Safety 94 (2), 611–617.

Jiang, R., Jardine, A. K. S., 2007. An optimal burn-in preventive-replacement model associated with a mixture distribution. Quality and Reliability Engineering International 23 (1), 83–93.

Khojandi, A., Maillart, L. M., Prokopyev, O. A., 2014. Optimal planning of life-depleting maintenance activities. IIE Transactions 46 (7), 636–652.

Lawless, J., 2002. Statistical Models and Methods for Lifetime Data, 2nd Edition. Wiley-Interscience.

Murthy, D. N. P., Iskandar, B. P., Wilson, R. J., 1995. Two-dimensional failure-free warranty policies: Two-dimensional point process models. Operations Research 43 (2), 356–366.

Park, M., Pham, H., 2012. A generalized block replacement policy for a k-out-of-n system with respect to threshold number of failed components and risk costs. IEEE Transactions on Systems, Man, and Cybernetics 42 (2), 453–463.

Resnick, S. I., 2013. Adventures in Stochastic Processes. Springer.

Rinne, H., 2008. The Weibull Distribution: A Handbook, 1st Edition. Chapmann and Hall/CRC.

Saassouh, B., Dieulle, L., Grall, A., 2007. Online maintenance policy for deteriorating system with random change of mode. Reliability Engineering and System Safety 92 (12), 1677–1685.

Sheu, S.-H., Zhang, Z. G., 2013. An optimal age replacement policy for multi-state systems. IEEE Transactions on Reliability 62 (3), 722–734.

Tinga, T., 2010. Application of physical failure models to enable usage and load based mainteance. Reliability Engineering and System Safety 95 (10), 1061–1075.

Xia, T., Jin, X., Xi, L., Ni, J., 2015. Production-driven opportunistic maintenance for batch production based on MAM-APB scheduling. European Journal of Operational Research 240 (3), 781–790.

Xie, M., 1989. On the solution of renewal-type integral equations. Communications in Statistics-Simulation and Computation 18 (1), 281–293.

Xu, W., Cao, L., 2015. Optimal tool replacement with product quality deterioration and random tool failure. International Journal of Production Research 53 (6), 1736–1745.

Zhang, X., Kang, J., Jin, T., 2014. Degradation modeling and maintenance decisions based on Bayesian belief networks. IEEE Transactions on Reliability 63 (2), 620–633.

Zhao, X., Mizutani, S., Nakagawa, T., 2015. Which is better for replacement policies with continuous or discrete scheduled times? European Journal of Operational Research 242, 477–486.

Zhou, B., Yu, J., Shao, J., Trentesaux, D., 2015. Bottleneck-based opportunistic maintenance model for series production systems. Journal of Quality in Maintenance Engineering 21 (1), 70–88.

Zitrou, A., Bedford, T., Daneshkhah, A., 2013. Robustness of maintenance decisions: Uncertainty modelling and value of information. Reliability Engineering and System Safety 120, 60–71.